Summer School on Mathematics of Shapes

(4 – 15 July 2016)

Venue:
IMS Auditorium
3 Prince George's Park, Singapore 118402

Tuesday, 5 July 2016	
08:55am – 09:00am	Registration
09:00am – 10:00am	Tutorial on Wavelets I *Hui Ji, National University of Singapore*
10:00am – 10:30am	*--- Group Photo and Coffee Break ---*
10:30am – 11:30am	Tutorial on Wavelets II *Hui Ji, National University of Singapore*
11:30am – 02:00pm	*--- Lunch Break ---*
02:00pm – 03:00pm	Tutorial on Wavelets III *Hui Ji, National University of Singapore*
03:00pm – 03:30pm	*--- Coffee Break ---*
03:30pm – 04:30pm	Tutorial on Wavelets IV *Hui Ji, National University of Singapore*
Wednesday, 6 July 2016	
--- Public Holiday: Office Closed ---	
Thursday, 7 July 2016	
08:55am – 09:00am	Registration
09:00am – 10:00am	Introduction to the Differential Geometry I *Sergey Kushnarev, Singapore University of Technology and Design*
10:00am – 10:30am	*--- Coffee Break ---*
10:30am – 11:30am	Introduction to the Differential Geometry II *Sergey Kushnarev, Singapore University of Technology and Design*
11:30am – 02:00pm	*--- Lunch Break ---*
02:00pm – 03:00pm	Introduction to the Differential Geometry III *Sergey Kushnarev, Singapore University of Technology and Design*
03:00pm – 03:30pm	*--- Coffee Break ---*
03:30pm – 04:30pm	Introduction to the Differential Geometry IV *Joan Alexis Glaunès, Université Paris Descartes, France*

Friday, 8 July 2016	
08:55am – 09:00am	Registration
09:00am – 10:00am	Introduction to the Differential Geometry V *Sergey Kushnarev, Singapore University of Technology and Design*
10:00am – 10:30am	*--- Coffee Break ---*
10:30am – 11:30am	Introduction to the Differential Geometry VI *Sergey Kushnarev, Singapore University of Technology and Design*
11:30am – 02:00pm	*--- Lunch Break ---*
02:00pm – 03:00pm	Introduction to the Differential Geometry VII *Sergey Kushnarev, Singapore University of Technology and Design*
03:00pm – 03:30pm	*--- Coffee Break ---*
03:30pm – 04:30pm	Introduction to the Differential Geometry VIII *Joan Alexis Glaunès, Université Paris Descartes, France*
Monday, 11 July 2016	
08:55am – 09:00am	Registration
09:00am – 10:00am	Diffeomorphic Models and Matching Problems in the Discrete Case *Joan Alexis Glaunès, Université Paris Descartes, France*
10:00am – 10:30am	*--- Coffee Break ---*
10:30am – 11:30am	Reproducing Kernels in the Vectorial Case *Joan Alexis Glaunès, Université Paris Descartes, France*
11:30am – 02:00pm	*--- Lunch Break ---*
02:00pm – 03:00pm	Tutorial on Manifolds of Diffeomorphisms, EPDiff I *Martins Bruveris, Brunel University London, UK*
03:00pm – 03:30pm	*--- Coffee Break ---*
03:30pm – 04:30pm	Tutorial on Manifolds of Diffeomorphisms, EPDiff II *Martins Bruveris, Brunel University London, UK*
Tuesday, 12 July 2016	
08:55am – 09:00am	Registration
09:00am – 10:00am	Geodesic Equations and Shooting Algorithms for Matching and Template Estimation *Joan Alexis Glaunès, Université Paris Descartes, France*
10:00am – 10:30am	*--- Coffee Break ---*
10:30am – 11:30am	Tutorial on Manifolds of Diffeomorphisms, EPDiff I *Martin Bauer, University of Vienna, Austria*
11:30am – 02:00pm	*--- Lunch Break ---*
02:00pm – 03:00pm	Tutorial on Manifolds of Diffeomorphisms, EPDiff II *Martin Bauer, University of Vienna, Austria*
03:00pm – 03:30pm	*--- Coffee Break ---*
03:30pm – 04:30pm	Tutorial on Manifolds of Diffeomorphisms, EPDiff III *Martins Bruveris, Brunel University London, UK*

Wednesday, 13 July 2016	
08:55am – 09:00am	Registration
09:00am – 10:00am	Models for Diffeomorphic Mappings between Submanifolds: Measures, Currents, Varifolds *Joan Alexis Glaunès, Université Paris Descartes, France*
10:00am – 10:30am	*--- Coffee Break ---*
10:30am – 11:30am	Tutorial on Manifolds of Diffeomorphisms, EPDiff III *Martin Bauer, University of Vienna, Austria*
11:30am – 02:00pm	*--- Lunch Break ---*
02:00pm – 03:00pm	Tutorial on Manifolds of Diffeomorphisms, EPDiff IV *Martins Bruveris, Brunel University London, UK*
03:00pm – 03:30pm	*--- Coffee Break ---*
03:30pm – 04:30pm	Tutorial on Statistics on Shape Spaces I *Tom Fletcher, University of Utah, USA*
Thursday, 14 July 2016	
08:55am – 09:00am	Registration
09:00am – 10:00am	Tutorial on Manifolds of Diffeomorphisms, EPDiff IV *Martin Bauer, University of Vienna, Austria*
10:00am – 10:30am	*--- Coffee Break ---*
10:30am – 11:30am	Lie Groups and Lie Group Actions I *Richard Hartley, Australian National University, Australia*
11:30am – 02:00pm	*--- Lunch Break ---*
02:00pm – 03:00pm	Lie Groups and Lie Group Actions II *Richard Hartley, Australian National University, Australia*
03:00pm – 03:30pm	*--- Coffee Break ---*
03:30pm – 04:30pm	Tutorial on Statistics on Shape Spaces II *Tom Fletcher, University of Utah, USA*
Friday, 15 July 2016	
08:55am – 09:00am	Registration
09:00am – 10:00am	Lie Groups and Lie Group Actions III *Richard Hartley, Australian National University, Australia*
10:00am – 10:30am	*--- Coffee Break ---*
10:30am – 11:30am	Lie Groups and Lie Group Actions IV *Richard Hartley, Australian National University, Australia*
11:30am – 02:00pm	*--- Lunch Break ---*
02:00pm – 03:00pm	Tutorial on Statistics on Shape Spaces III *Tom Fletcher, University of Utah, USA*
03:00pm – 03:30pm	*--- Coffee Break ---*
03:30pm – 04:30pm	Tutorial on Statistics on Shape Spaces IV *Tom Fletcher, University of Utah, USA*

Tutorial on Manifolds of Diffeomorphisms, EPDiff

Martin Bauer, University of Vienna, Austria

1) Riemannian geometries on the space of curves I
2) Riemannian geometries on the space of curves II

Abstract (1) and (2): The space of curves is of importance in the field of shape analysis. I will provide an overview of various Riemannian metrics that can be defined thereon, and what is known about the properties of these metrics. I will put particular emphasis on the induced geodesic distance, the geodesic equation and its well-posedness, geodesic and metric completeness and properties of the curvature. In addition I will present selected numerical examples illustrating the behaviour of these metrics.

3) Right invariant metrics on the diffeomorphism group

The interest in right invariant metrics on the diffeomorphism group is fuelled by its relations to hydrodynamics. Arnold noted in 1966 that Euler's equations, which govern the motion of ideal, incompressible fluids, can be interpreted as geodesic equations on the group of volume preserving diffeomorphisms with respect to a suitable Riemannian metric. Since then other PDEs arising in physics have been interpreted as geodesic equations on the diffeomorphism group or related spaces. Examples include Burgers' equation, the KdV and Camassa-Holm equations or the Hunter-Saxton equation.

Another important motivation for the study of the diffeomorphism group can be found in its appearance in the field of computational anatomy and image matching: the space of medical images is acted upon by the diffeomorphism group and differences between images are encoded by diffeomorphisms in the spirit of Grenander's pattern theory. The study of anatomical shapes can be thus reduced to the study of the diffeomorphism group.

Using these observations as a starting point, I will consider the class of Sobolev type metrics on the diffeomorphism group of a general manifold M. I will discuss the local and global well-posedness of the corresponding geodesic equation, study the induced geodesic distance and present selected numerical examples of minimizing geodesics.

4) The space of densities

I will discuss various Riemannian metrics on the space of densities. Among them is the Fisher--Rao metric, which is of importance in the field of information geometry. Restricted to finite-dimensional submanifolds, so-called statistical manifolds, it is called Fisher's information metric. The Fisher--Rao metric has the property that it is invariant under the action of the diffeomorphism group. I will show, that on a closed manifold of dimension greater than one, every smooth weak Riemannian metric on the space of smooth positive probability densities, that is invariant under the action of the diffeomorphism group, is a multiple of the Fisher--Rao metric.

Tutorial on Manifolds of Diffeomorphisms, EPDiff

Martins Bruveris, Brunel University London, UK

Lecture I - Mapping spaces as manifolds

This lecture will give an introduction to differential geometry in infinite dimensions. The main objects of shape analysis - the diffeomorphism group, the spaces of curves, surfaces, densities - can all be modelled as infinite-dimensional manifolds.

Lecture II - Riemannian geometry in infinite dimensions

Parts of Riemannian geometry generalise easily from finite to infinite dimensions. These include the definition of metric, covariant derivative, geodesic equations and curvature. But there are also qualitative differences, in particular with the distinction between strong and weak Riemannian metrics. This lecture will show some of the purely behaviour that can be encountered in infinite dimensions.

Lectures III and IV - Riemannian metrics induced by the diffeomorphism group

The purpose of these lectures is to explore the geometry of Riemannian metrics on the space of curves and landmarks that are induced by the action of the diffeomorphism group. These metrics correspond to exact matching of curves and landmarks via LDDMM. We will look at the induced metrics, geodesic equations and the geodesic distance.

Introduction to the Differential Geometry

Joan Alexis Glaunès (Université Paris Descartes, France) and
Sergey Kushnarev (Singapore University of Technology and Design)

1. Definition of a manifold, Tangent Vectors and Tangents Spaces, Pushforwards, Vector Fields.
2. Tangent bundle and a Cotangent Bundle, Pullbacks, Tensors, Differential Forms.
3. Submersions, Immersions, Embeddings, Submanifolds (Embedded, Immersed)
4. Integral Curves and Flows, Lie Derivatives.
5. Riemannian Metrics.
6. Connections.
7. Riemannian Geodesics and Distance (exp map, normal coordinates, geodesics and minimizing distances).
8. Curvature.

Diffeomorphic Models and Matching Problems in the Discrete Case

Joan Alexis Glaunès, Université Paris Descartes, France

This talk will be an introduction and on overview of the framework of diffeomorphic mappings (LDDMM) for estimating deformations between shapes, and its formulation for discrete problems via reproducing kernels. I will present the classical construction of the group of diffeomorphisms, and explain how by considering different types of actions on this group, it can be used to estimate deformations between different types of geometric data: images, points, surfaces, etc. I will show some experiments and studies to illustrate.

Geodesic Equations and Shooting Algorithms for Matching and Template Estimation

Joan Alexis Glaunès, Université Paris Descartes, France

In this talk I will explain the link between diffeomorphic mappings and shape spaces, i.e. Riemannian metrics on sets of shapes. I will explain how the metric on the group of diffeomorphisms induces a metric on the space of shapes, and detail the geodesic equations in the finite dimensional case (manifold of landmarks), which is the case in use in practice for many problems once data has been discretised. I will present different algorithms which are based on these equations (geodesic shooting algorithms): matching, template estimation, geodesic regression, and explain how all this can be actually implemented.

Models for Diffeomorphic Mappings between Submanifolds: measures, currents, varifolds

Joan Alexis Glaunès, Université Paris Descartes, France

This talk will focus on some models for defining data attachment terms for matching problems between submanifolds (curves or surfaces) which are widely used for diffeomorphic mappings. These are all based on the same idea of defining dual RKHS spaces and using the corresponding norm as a data attachment term between shapes. This uses mathematical concepts such as currents or varifolds, which come from geometric measure theory and which I will introduce. I will present both continuous and discrete forms of these models, and show some outputs of algorithms

Reproducing Kernels in the Vectorial Case

Joan Alexis Glaunès, Université Paris Descartes, France

The theory of reproducing kernels and Reproducing Kernel Hilbert Spaces (RKHS) is extensively used in the discrete formulation of the LDDMM setting, and in corresponding algorithms. It is also a fundamental concept in other areas, such as statistical learning. I will present some basic concepts of this theory in the general case of RKHS of vector fields, and explain how this theory can be used for interpolation problems, and how it is linked to the LDDMM setting. I will also present shortly a recent study about translation and rotation invariant kernels, which allows in particular to consider spaces of divergence free or irrotational vector fields for deformation analysis.

Lie Groups and Lie Group Actions

Richard Hartley, Australian National University, Australia

I will talk about Lie groups and Lie group actions on manifolds, with particular consideration for applications in Computer Vision. Lie groups play a significant role in Computer Vision, particularly groups such as SO(3), the group of 3-D rotations, SE(3), the group of Euclidean motions, and PGL(2,R) and PGL(3, R), the groups of 2 and 3-dimensional projective transformations. In addition, Lie group actions on such manifolds as the Stiefel manifolds (yielding Grassman manifolds as a space of orbits) and the action of O(2) on SO(3) x SO(3), yielding the Essential manifold, as well as Shape manifolds as an orbit space of an action of similarity transforms, are common examples where Lie group actions give rise to Riemannian manifold structures. Applications are in the areas of Lie group tracking, averaging (for instance rotation averaging), and kernels on manifolds such as shape manifolds and Grassman manifolds, all with important applications in computer vision and robotic vision.

Tutorial on Wavelets

Hui Ji, National University of Singapore

This lecture focuses on the introduction to wavelet frame and its applications in imaging and vision. The goal to expose audience to important topics in wavelet frames with strong relevance to visual data processing, in particular image processing/analysis. The audience will also learn how to apply these methods to solve real problems in imaging and vision. The lecture is an inter-disciplinary one that emphasizes both rigorous treatment in mathematics and motivations from real-world applications.