Color Histograms using the perceptual metric

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Plan of the presentation

- Formalization of the notion of image histogram
- Perceptual metric and Macadam ellipses
- Density estimation in the space of colors

Image histogram : formalization

$$I : \begin{cases} \Omega & \rightarrow & V \\ p & \mapsto & I(p) \end{cases}$$

- Ω: support space of pixels: rectangle/parallelepiped.
- V: the value space

(Ω, μ_Ω), (V, μ_V), μ_Ω and μ_V are induced by the choosen geometries on Ω and V.

Transport of μ_Ω on V: $I^*(\mu_\Omega)$

Image histogram: estimation of

$$f = \frac{dI^*(\mu_\Omega)}{d\mu_V}$$

pixels: $p \in \Omega$, uniformly distributed with respect to μ_Ω

$\{I(p), p$ a pixel $\}$: set of independent draws of the "random variable" I

Estimation of $f = \frac{dI^*(\mu_\Omega)}{d\mu_V}$ from $\{I(p), p$ a pixel $\}$:
$\rightarrow$ standard problem of probability density estimation

Perceptual color histograms

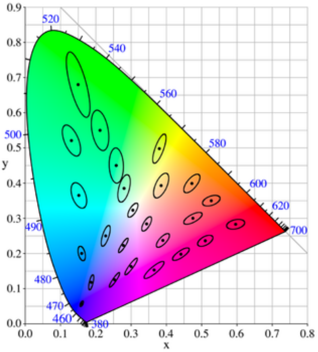
$$I : \left\{ \begin{array}{ccc} \Omega & \rightarrow & (\mathcal{M} = colors, g_{perceptual}) \\ p & \mapsto & I(p) \end{array} \right.$$

Assumption: the perceptual distances between colors is induced by a Riemannian metric

The manifold of colors was one of the first example of Riemannian manifold, suggested by Riemann

Macadam ellipses: just noticeable differences

Chromaticity diagram (constant luminance):



Ellipses: elementary unit balls $\rightarrow$ local L^2 metric

Lab space

The Euclidean metric of the *Lab* parametrization is supposed to be more perceptual than other parametrizations

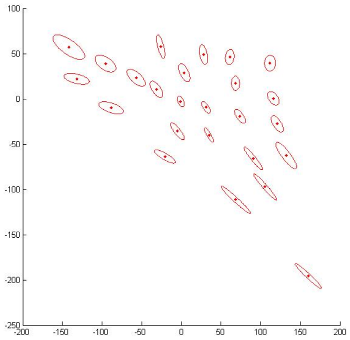


Figure: Macadam ellipses in the *ab* plan

However, the ellipses are clearly not balls

Modifiction of the density estimator

Density $\rightarrow$ local notion. No need of knowing long geodesics
Small distances $\rightarrow$ local approximation by an Euclidean metric

Notations:
d_R: Perceptual metric
$||.||_{Lab}$: Canonical Euclidean metric of Lab
$||.||_c$: Euclidean metric on Lab induced by the ellipse at c

Small distances around c: $||.||_c$ is "better" than $||.||_{Lab}$

Modifiction of the density estimator

Standard kernel estimator:

$$\hat{f}(x) = \frac{1}{k} \sum_{p_i \in \{pixels\}} \frac{1}{r^2} K\left(\frac{||x - I(p_i)||_{Lab}}{r}\right)$$

Possible modification

$$K\left(\frac{||x - I(p_i)||_{Lab}}{r}\right) \to K\left(\frac{||x - I(p_i)||_{I(p_i)}}{r}\right)$$

where $||.||_{I(p_i)}$ is an Euclidean distance defined by the interpolated ellipse at $I(p_i)$.

Generally, at c a color:

$$lim_{x\to c}\frac{||x-c||_c}{d_R(x,c)} = 1 \neq lim_{x\to c}\frac{||x-c||_{Lab}}{d_R(x,c)}$$

Thus, $\exists A > 0$ such that,

$$\forall R > 0, \exists x \in B_{Lab}(c,R), A < \left|\frac{||x-c||}{d_R(x,c)} - 1\right|.$$

while $\exists R_c = R_{c,A}$ such that,

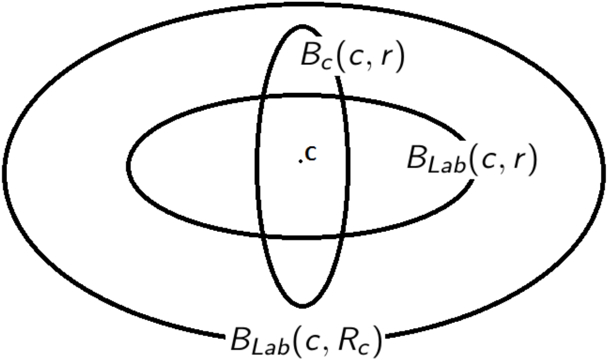
$$\forall x \in B_{Lab}(c,R_c), \left|\frac{||x-c||_c}{d_R(x,c)} - 1\right| < A.$$

hence

$$sup_{B_{Lab}(c,R_c)}\left(\left|\frac{||x-c||_c}{d_R(x,c)} - 1\right|\right) < A < sup_{B_{Lab}(c,R_c)}\left(\left|\frac{||x-c||}{d_R(x,c)} - 1\right|\right).$$

When the scaling factor r is small enough:

$$r \leq R_c \text{ and } B_c(c,r) \subset B_{Lab}(c,R_c)$$



- $x \in B(c,R_c)$, $K\left(\frac{||x-c||_c}{r}\right)$ better than $K\left(\frac{||x-c||_{Lab}}{r}\right)$.
- $x \notin B(c,R_c)$, $K\left(\frac{||x-c||_c}{r}\right) = K\left(\frac{||x-c||_{Lab}}{r}\right) = 0$

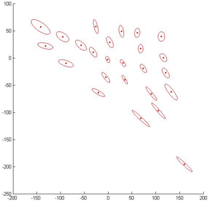
Interpolation of a set of local metric: a deep question...

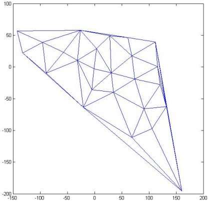
What is a good interpolation?

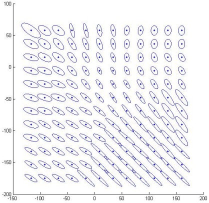
- Interpolating a function: minimizing variation with respect to a metric.
- Interpolating a metric? No intrinsic method: depends on a choice of parametrization.

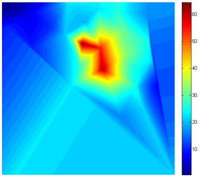
Subject of the next study

Barycentric interpolation in the *Lab* space





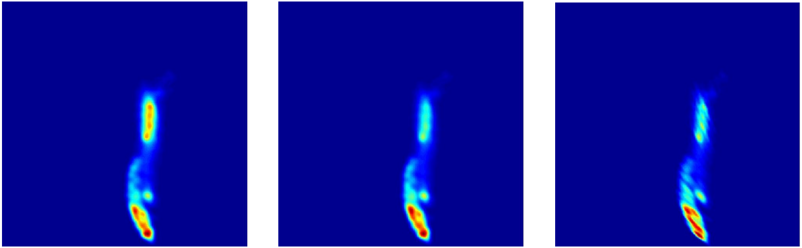




Volume change

(a) (b)

Figure: (a): color photography (b): Zoom of the density change adapted to colours present in the photography



(a) (b) (c)

Figure: The canonical Euclidean metric of the *ab* projective plane in (a), the canonical metric followed by a division by the local density of the perceptual metric in (b) and the modified kernel formula in (c).

Conclusion

- A simple observation which improve the consistency of the histogram without requiring additional computational costs
- Future works will focus on:
  - The interpolation of the ellipses
  - The construction of the geodesics and their applications

Thank you for your attention